Logics of Truthmaker Semantics: Comparison, Compactness and Decidability

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University of Amsterdam

Outline of the talk

- Context, motivation and general aim
- Defining the truthmaker framework
- Presenting proof (outlines) of formal properties of ‘truthmaker logics’
- Conclusion

Context and general aim

Background

- (Finean) truthmaker semantics (TS) was introduced to model 'exact truthmaking'.
- Great interest in TS as a framework for analyzing various philosophical and linguistic phenomena, e.g., metaphysical grounding, counterfactuals and implicatures [cf. Fine (2017c)].
- But limited study of the various logics arising from the semantics [exceptions being Fine and Jago (2019) and Korbmacher (2022)].

This talk aims to address this gap by exploring numerous 'truthmaker logics'

In particular:

1. Translations and Compactness
2. Finite Model Property (FMP) and Decidability
3. Connection with modal (information) logic [will perhaps be skipped]

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But why be interested in the *logics* of truthmaker semantics?

Motivation

Why *logics* of truthmaker semantics?

Several philosophical concepts find expression as consequence or equivalence within a truthmaker logic:

- According to Jago (2017), both samesaying of sentences and identity of propositions amount to truthmaker equivalence.
- As studied by Fine (2017a,b), notions of ground and of containment can be captured by truthmaker consequence.[1]

But primarily, motivated by logical curiosity:

- Is truthmaker consequence compact? I.e., determined by behaviour on *finite* sets of formulae.
- Is truthmaker consequence decidable?
- And can we develop something like a truthmaker analogue of the FMP?
- Do the answers to these questions vary across the truthmaker logics?
- And even if not, which – if any – of these logics coincide?

[1](i) P weakly grounds Q iff P truthmaker entails Q;

(ii) P weakly partially grounds Q iff $(P \wedge Q) \vee Q$ is truthmaker equivalent to Q; and

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I’ve now discussed why it’s worthwhile to study the metalogic of truthmaking [and hopefully convinced you in the process ^^] … but what even is truthmaker semantics?

Defining truthmaker semantics

Definition (language and semantics)

The language is given by

$$\varphi ::= \; p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi.$$

The semantics are *bilateral* (truthmaking $\Vdash^+$ and falsitymaking $\Vdash^-$), and models come with two valuations V^+, V^-:

$$\mathbb{M}, s \Vdash^{\pm} p \quad \text{iff} \quad s \in V^{\pm}(p).$$
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Example

How about '∨' and falsitymaking '∧'?

Truthmaker framework: *Semantics* parameter 1

Non-incl.: $\mathbb{M}, s \Vdash^+ \varphi \vee \psi$ iff $\mathbb{M}, s \Vdash^+ \varphi$ *or* $\mathbb{M}, s \Vdash^+ \psi$.

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Falsitymaking $\Vdash^-$ of '∨' and '∧' are defined by de Morgan's.

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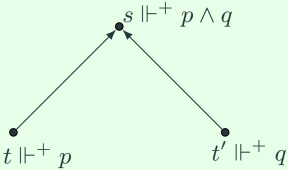

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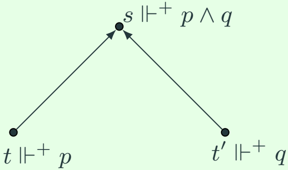

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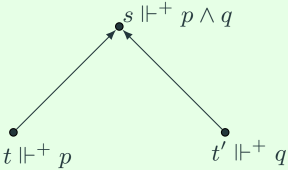

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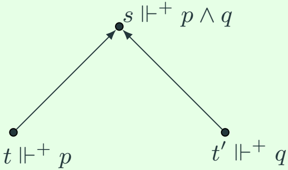

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Convex truthmaking

Non-convexity

The presented semantics allow for *non-convex* truthmaking:

cases where $r \leq s \leq t, \mathbb{M}, r \Vdash^{+} \varphi$ and $\mathbb{M}, t \Vdash^{+} \varphi,$ but $\mathbb{M}, s \nVdash^{+} \varphi$.

To avoid this,[2] we can define convex truth- and falsitymaking:

Truthmaker framework: *Semantics* parameter 2

Convex:	$\mathbb{M}, s \Vdash^{\pm,c} \varphi$	:iff	$\exists r, t \in S$ such that $\mathbb{M}, r \Vdash^{\pm} \varphi,\ \mathbb{M}, t \Vdash^{\pm} \varphi,$ and $r \leq s \leq t$.
'Non-convex':	$\mathbb{M}, s \Vdash^{\pm} \varphi$	iff	$\mathbb{M}, s \Vdash^{\pm} \varphi$.

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Truthmaker framework: *Valuation* parameter

- **All:** Any pairs of valuations $V^{\pm} : \mathbf{P} \to \mathcal{P}(S)$ are admissible.
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Truthmaker framework: *Frame* parameter

$\mathcal{S}_1 := \{(S, \leq) \mid (S, \leq)$ *is a semilattice*$\}$.

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(so $\mathcal{C}_2$ is the class of complete lattices.)

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For any choice of semantics, valuations and frames, we get a *truthmaker consequence relation* by defining

$\Gamma \Vdash^{+} \varphi$:iff whenever $\mathbb{M}, s \Vdash^{+} \gamma$ for all $\gamma \in \Gamma$, it is also the case that $\mathbb{M}, s \Vdash^{+} \varphi$

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- **All:** Any pairs of valuations $V^{\pm} : \mathbf{P} \to \mathcal{P}(S)$ are admissible.
- **Closure under binary joins:** if $\{s, t\} \subseteq V^{\pm}(p)$, then $\sup\{s, t\} \in V^{\pm}(p)$.
- **Non-vacuity:** $V^{+}(p) \neq \varnothing$ for all $p \in \mathbf{P}$ and/or $V^{-}(p) \neq \varnothing$ for all $p \in \mathbf{P}$.

Truthmaker framework: *Frame* parameter

$$\mathcal{S}_1 := \{(S, \leq) \mid (S, \leq) \textit{ is a semilattice}\}.$$
$$\mathcal{S}_2 := \{(S, \leq) \mid (S, \leq) \textit{ is a semilattice with a bottom element}\}.$$
$$\mathcal{C}_1 := \{(S, \leq) \mid (S, \leq) \textit{ is a poset with all non-empty joins}\}.$$
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(so $\mathcal{C}_2$ is the class of complete lattices.)

Truthmaker logics

For any choice of semantics, valuations and frames, we get a *truthmaker consequence relation* by defining

$$\Gamma \Vdash^{+} \varphi \text{ :iff whenever } \mathbb{M}, s \Vdash^{+} \gamma \text{ for all } \gamma \in \Gamma, \text{ it is also the case that } \mathbb{M}, s \Vdash^{+} \varphi$$

So we have (at least) $2 \times 2 \times 4 \times 4 = 64$ logics to survey $\ldots$

Luckily, they can be dealt with (rather) uniformly!

Proof strategy

Proceeding from here, our proof strategy is as follows:

1. Inherit compactness and recursive enumerability from first-order logic through translations *for* ***semilattice*** *truthmaker logics*.
2. Develop and prove a truthmaker analogue of the finite model property to obtain decidability *for semilattice truthmaker logics*.
3. Show that truthmaker consequence is invariant for choice of frames, which also entails that 'all' truthmaker logics are (compact and) decidable.

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Translations into first-order logic

Definition (translation into FOL)

We define the following translation-pair into first-order logic (FOL):

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\begin{aligned}
ST_x^+(p) &= P^T x \\
ST_x^-(p) &= P^F x \\
ST_x^+(\neg\phi) &= ST_x^-(\phi) \\
ST_x^-(\neg\phi) &= ST_x^+(\phi) \\
ST_x^+(\phi \wedge \psi) &= \exists y, z\big(x = \sup\{y, z\} \wedge ST_y^+(\phi) \wedge ST_z^+(\psi)\big) \\
ST_x^-(\phi \wedge \psi) &= ST_x^-(\phi) \vee ST_x^-(\psi) \\
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\end{aligned}
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Proposition (correspondence)

For all models $\mathbb{M}$ and all $\varphi \in \mathcal{L}_T$, we have:

For all states $s \in \mathbb{M}$: (i) $\mathbb{M}, s \Vdash^+ \varphi$ iff $\mathbb{M} \vDash ST_x^+(\varphi)[s]$; *and*

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Compactness and recursive enumerability

Proposition (semilattice compactness and r.e.)

All semilattice truthmaker logics are

- **compact:** *if* $\Gamma \Vdash^+ \varphi$, *then* $\Gamma_F \Vdash^+ \varphi$ for some finite $\Gamma_F \subseteq \Gamma$; and
- **r.e.:** For finite Γ_F, we can effectively enumerate (Γ_F, φ) s.t. $\Gamma_F \Vdash^+ \varphi$.

Proof.

Let J be the first-order formula defining (join-)semilattices. For **compactness**, the argument is essentially that

$$\Gamma \Vdash^+ \varphi \quad \text{iff} \quad ST^+_x(\Gamma) \cup \{J\} \vDash ST^+_x(\varphi)$$

$$\overset{(c)}{\text{iff}} \quad ST^+_x(\Gamma_F) \cup \{J\} \vDash ST^+_x(\varphi) \quad \text{iff} \quad \Gamma_F \Vdash^+ \varphi,$$

where $\Gamma_F \subseteq \Gamma$ is finite.
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$$\Gamma_F \Vdash^+ \varphi \quad \text{iff} \quad ST^+_x(\Gamma_F) \cup \{J\} \vDash ST^+_x(\varphi)$$

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□

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FMP and decidability

Initial observation: A direct analogue of, e.g., the modal logic FMP is trivial and does nothing for proving decidability. Instead, we prove the following:

Theorem (Truthmaker FMP)

For any model $\mathbb{M}_0 = (S_0, \leq_0, V_0^+, V_0^-)$, state $s \in S_0$, and finite set of formulas $\Gamma_F \subseteq \mathcal{L}_T$ s.t. $\mathbb{M}_0, s \Vdash^+ \Gamma_F$,
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What about other classes of frames?

Limitation of translation method: it only applies when conditions are first-order definable. And having, e.g., all joins is not.

Second-order frames

Definition (recall)

$$\mathcal{S}_1 := \{(S, \leq) \mid (S, \leq) \text{ is a semilattice}\},$$
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Theorem (Entailment Invariance for Choice of Frames)

Given any choice of semantics and valuations, and any $X, Y \in \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{C}_1, \mathcal{C}_2\}$,

$$\Gamma \Vdash^+_X \varphi \quad \text{iff} \quad \Gamma \Vdash^+_Y \varphi.$$

Proof idea.

Clearly, $\Gamma \Vdash^+_{\mathcal{S}_1} \varphi \Rightarrow \Gamma \Vdash^+_{\mathcal{S}_2/\mathcal{C}_1} \varphi \Rightarrow \Gamma \Vdash^+_{\mathcal{C}_2} \varphi.$

Therefore, $\Gamma \Vdash^+_{\mathcal{S}_1} \varphi \Leftarrow \Gamma \Vdash^+_{\mathcal{C}_2} \varphi$ suffices, which is a consequence of our 'Completion Lemma' showing how to complete a semilattice into a complete lattice in a satisfaction-preserving and -reflecting way. □

Corollary (compactness and decidability)

'All' truthmaker logics are compact and decidable.

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Corollary (compactness and decidability)

'All' truthmaker logics are compact and decidable.

Second-order frames

Definition (recall)

$$\mathcal{S}_1 := \{(S, \leq) \mid (S, \leq) \text{ is a semilattice}\},$$
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Theorem (Entailment Invariance for Choice of Frames)

Given any choice of semantics and valuations, and any $X, Y \in \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{C}_1, \mathcal{C}_2\}$,

$$\Gamma \Vdash^+_X \varphi \quad \text{iff} \quad \Gamma \Vdash^+_Y \varphi.$$

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Completion Lemma

Lemma

Let $(S, \leq)$ be a semilattice and $\mathcal{U}(S) \subseteq \mathcal{P}(S)$ its collection of upsets. Then (i) $(\mathcal{U}(S), \supseteq)$ forms a complete lattice, and (ii) for all $s, t, u \in S$:

$$s = \sup{}_{\leq}\{t, u\} \qquad \text{iff} \qquad {\uparrow}s = {\uparrow}t \cap {\uparrow}u.$$

Lemma

For all formulas $\varphi \in \mathcal{L}_T$ and $\mathbb{M}, s$ s.t. $\mathbb{M}, s \Vdash^+ \varphi$, there are literals $l_1, \ldots l_n$ s.t.

1. $(l_1 \wedge \cdots \wedge l_n) \Vdash^+_{\mathcal{S}_1} \varphi$,
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Conclusion

Summary:

- Translated into FOL, achieving r.e. and compactness (for some truthmaker logics).
- Developed and proved the FMP, achieving decidability (for some truthmaker logics).
- Showed that truthmaker consequence is invariant for choice of frames, allowing us to additionally conclude that 'all' truthmaker logics are compact and decidable.
- Gave a modal perspective on truthmaker semantics: adding Boolean negation results in modal information logics.

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Thank you sincerely for attending, even on a holiday :-)

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Our proof techniques bear a resemblance to modal logic. Can we elucidate and precisify this resemblance?

A modal perspective on truthmaker semantics

Definition (van Benthem (2019)'s translation)

Let $\mathcal{L}_M$ be the language of modal information logic; i.e., the modal language with a single binary modality '$\langle \mathrm{sup} \rangle$' (for supremum). Define the following translation:

$$
\begin{aligned}
(p)^+ &= p^T, & (p)^- &= p^F,\\
(\neg\varphi)^+ &= \varphi^-, & (\neg\varphi)^- &= \varphi^+,\\
(\varphi \wedge \psi)^+ &= \langle \mathrm{sup} \rangle \varphi^+ \psi^+, & (\varphi \wedge \psi)^- &= \varphi^- \vee \psi^-,\\
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Definition

Let $\mathcal{L}_M^{\{p^T, p^F, \vee, \langle \mathrm{sup} \rangle\}} \subseteq \mathcal{L}_M$ be the fragment of the language of modal information logic restricted to the propositional letters, connective '$\vee$' and modality '$\langle \mathrm{sup} \rangle$'. Define the following translation:

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(p^T)^\bullet &= p, & (p^F)^\bullet &= \neg p,\\
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A modal perspective on truthmaker semantics (continued)

Proposition

The translations $(\cdot)^+$ and $(\cdot)^\bullet$ are each other's 'inverses':

For all $\varphi \in \mathcal{L}_T$ *and all* $\mathbb{M}, s$*:* $\mathbb{M}, s \Vdash^+ \varphi$ **iff** $\mathbb{M}, s \Vdash^+ (\varphi^+)^\bullet$.

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Corollary (Characterization)

Truthmaker logics are (in a precise mathematical sense) the $\{\vee, \langle \mathrm{sup} \rangle\}$-fragments of modal information logics, or alternatively, modal information logics arise from augmenting truthmaker logics with classical negation.

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